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# Comparing the performance of Median or Mean and other risk indicators in Portfolio Optimization

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## EXTENDED ABSTRACT

### INTRODUCTION

Markowitz model of asset portfolio optimization has some deficits, one of the most important of which is the normality assumption of stock market returns. Normality of returns has been rejected in numerous studies and has been shown that mean is not a good maximization objective anymore. Mean of returns is quite sensitive to outliers. There are three different ways of facing this problem in the literature. The first approach is generally to abandon the modern portfolio theory and turn to using meta-heuristic algorithms in portfolio optimization. The second approach still considers the modern portfolio theory important and valuable and uses it with adjustments. Some studies under the ultra-modern portfolio theory have focused on the inappropriateness of variance and use other measures of risk. Some others studies focus on drastic changes in the optimal portfolio as a result of changes in input values and use robust statistics. The third approach, on the other hand, tries to avoid the mentioned problems simply by using different parameters in optimization instead of average and variance. This paper based on the third approach seeks to use the median instead of the mean in stock portfolio optimization. The purpose is to compare the performance of mean and median in optimization. Moreover, the variance is not enough to control the risk because of heavy tails of return distribution and, thus, this paper incorporates various risk measures into models to test which one performs better beside median as an alternative to mean-variance models.

### METHODOLOGY

Five median maximization models are presented with different risk measures of mean absolute deviations (MAD), value at risk (VaR), average value at risk (CVaR), and maximum loss (ML). Models are solved using GAMS software package and daily real data of twenty stocks from Tehran Stock Exchange from the beginning of 2016 to the end of 2019. For this purpose, the models are first employed for portfolio optimization in a certain period of 50 days. After computation, then the optimal weights are used for the next

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period of 50 days. This procedure is then repeated for the next 100 days to the end of 2019. Finally, the average and the distribution of returns of the optimal portfolio obtained from different models are compared with three other models: the mean optimization method without any control for risk, the mean optimization method subject to value at risk (VaR) constraint, and the portfolio with equal weights (EqW).

**FINDINGS**

Findings show that the median has a better performance in portfolio optimization. The model of median maximization gains higher returns in seventy percent of cases and a higher return on average. This means that a higher value of portfolio would be obtained using median in optimization. As a second conclusion, it was also shown that the median optimization method results in a portfolio with higher degree of diversification. The result remains true adding various risk measures to the model showing that median optimization instead of mean obtains a more diversified portfolio. In comparison among different median optimization models, it was also shown that CVaR and MAD risk measures controls the risk better than VaR and Maximum loss and obtains even further diversification.

## CONCLUSION

The results generally show that portfolio optimization models based on the mean instead of the median and subject to risk measures of the average value at risk (VaR) and the mean absolute deviations (MAD) have a better performance in return maximization, control of risk and portfolio diversification.

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