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On The Behavior of Malaysian Equities: Fractal Analysis Approach

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Abstract:
Fractal analyzing of continuous processes have recently emerged in literatures in various domains. Existence of long memory in many processes including financial time series has been evidenced via different methodologies in many literatures in the past decade. This has inspired many recent literatures on quantifying the fractional Brownian motion (fBm) characteristics of financial time series. This paper questions the accuracy of commonly applied fractal analyzing methods on explaining persistent or anti-persistent behavior of time series understudy. Rescaled range (R/S) and power spectrum techniques produce fractal dimensions for daily returns of twelve Malaysian stocks from the most well performed firms in Kuala Lumpur stock exchange. Zipf's law generates linear and logarithmic power-law distribution plots to evaluate the validity of estimated fractal dimensions on prescribing persistent and anti-persistent characteristics with less ambiguity. Findings of this study recommend a more thoughtful approach on classifying persistent and anti-persistent behaviors of financial time series by utilizing existing fractal analyzing methods.

JEL classification**:** C01, C02, C13, C61

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1. Introduction

here have been two major approaches in literatures facing financial time series since the last decades. Some literatures have considered a random unpredictable behavior for market and consequently have applied ready-to-use statistical methods, which are based on Gaussian distribution and Brownian motion, for explaining the market behavior. These literatures have used a vast number of statistical methods to examine market data, but mostly have applied the methods on a particular cluster of data without extending and examining the same methods on different scales. Other literatures have rejected a Gaussian behavior for market through observed non-stationary features and fat tails distribution of financial data.

Quantifying financial time series by considering their scale invariance, non-stationary and non-Gaussian behavior flourished following a series of valuable literatures by Mandelbrot [1]-[3] who created and developed the concept of fractal geometry and then the fractal dimension by extending the previous works of Hurst and Hölder [4],[5]. Considering scale invariance feature of financial time series, mining it with Fractal dimension may produce valuable insight for pattern recognition, modeling and forecasting of market behavior, overcoming the shortcomings of commonly used Gaussian based statistical methods.

Non-Gaussian distribution and long memory existence of market data have been observed and reported through many literatures [6],[7]. This fact has encouraged many researchers to apply new scale invariance methodologies based on fractal dimension and Hurst exponent to analyze market patterns. Many literatures have applied fractal analysis extensively on equities [8], [9], commodities [10] and exchange rates [11], [12]. Yet, complexity in measuring fractal dimension for one dimensional data (time series) has produced ambiguous results in the literatures. The common drive among majority of studies in this domain is to prove the existence of long memory in time series, then to evaluate its level of roughness through fractal dimension, and finally to show the persistence or anti-persistence characteristics of the series [13].

Uncertainty exists in measuring the exact amount of fractal dimension for various market data. Application of fractal dimension as an indicator to show the level of roughness, that is the level of volatility in financial time series [14], shows the significance of an accurate measurement for fractal dimension. In the literatures are rooted when applying different methods to extract the fractal dimension. Meanwhile, the coherence between a non-stationary fractional Brownian motion (*fBm*) and its counterpart, the fractional Gaussian noise (*fGn*), encouraged a new structured methodology of fractal analysis in[15], [16] which provide consistent results by applying most of fractal dimension estimation methods on a given class (*fBm* or *fGn*) and inconsistent results for the other class.

This paper reviews and examines two commonly used methods of measuring fractal dimension of time series; the Rescaled Range (*R/S*) analysis and Power Spectrum analysis. We use daily return data of 12 major stocks in Kuala Lumpur stock exchange (KLSE). This paper evaluates the accuracy of results produced by the two methods qualitatively through observational assessment with Zipf's law distribution of data.

2. Methodology

Fractal dimension and Hurst exponent

Financial time series are in the category of self-affine data. A self-affine set of data has a specific scale invariance feature in that different parts should be rescaled by different amounts in different directions to resemble the original. Self-affine fractals look more complicated than self-similar fractals, which are rescaled by the same factor in any direction and produce a more obvious scale invariance and likeness in different scales. Yet, the constant relation between scaling factors in self-affine fractals keep them scale invariant, which is the case of financial time series.

This scale invariant relationship is generally defined as:

$$SF_y = SF_x^H \quad (1)$$

where SF_x is the scaling factor in *X* coordinate; that is the time coordinate for time series data, SF_y is the scaling factor in *Y* coordinate; that is the return of market and *H* is the Hurst exponent.

Therefore, from the (1):

$$H = \log SF_y / \log SF_x \quad (2)$$

Hurst exponent has a range of zero to 1 and for self-affine set of data it is directly related to fractal dimension through:

$$D = 2 - H \quad (3)$$

where *D* is the fractal dimension and its range is between 1 and 2. In general, if in any set of data with $H=0.5(D=1.5)$ we have a Brownian motion, it suggests no correlation of data in *Y* coordinates (no correlation in market returns) although scaling factor in *Y* coordinate is related to scaling factor in *X* coordinate through a power-law rule. Characteristics of the self-affine fractal when $H\neq0.5(D\neq1.5)$ is the condition that has resulted in major ambiguity in the literatures. A time series has a “persistent” behavior for $0.5<H<(1\ (1<D<1.5)$, where changes in *Y* coordinate are positively correlated (that is, an increase is more expected to follow by another increase and vice versa). For “anti-persistent” behavior, $0<H<(0.5\ (1.5<D<2)$, we expect changes in *Y* are followed by a change in different directions.

Fractional Brownian motion (fBm) and Fractional Gaussian noise (fGn)

Mandelbrot [17] introduced the concept of "fractional Brownian motion (*fBm*)" to describe the processes characterized by $H\neq0.5(D\neq1.5)$ in contrast to a Brownian motion. There is another group of self-affine fractal processes called "fractional Gaussian noise *(fGn)*", that is a series of successive increments in an *fBm*. An *fGn* constitutes by applying a "First difference" transformation on an fBm and reversely applying a "Cumulative Sum" on an *fBm* constructs an *fGn*. Therefore, these two families of series are inter-convertible and carry the same Hurst exponent and Fractal dimension. However, they possess fundamentally different properties. The *fBm* process is non- stationary with stationary increments while the *fGn* is a stationary process with constant mean and variance.

Fractal Dimension estimation through Rescaled Range analysis

Rescaled Range method (*R/S*) was initiated by Hurst [5] for calculating the *H* exponent. For various windows *W* of a process, the range of observations *R(W)* in Y coordinate is divided by standard deviation of differenced data within same window *S(W)*, giving the *R(W)/S(W)* ratio. Hurst found a power-law relationship between *R(W)/S(W)* ratio and the sizes of observation windows, $R(W)/S(W)\ \alpha\ W^{H}$, where *H* is the Hurst exponent.

In practice, the series is divided into a number of intervals of length *w*, and *(w), S(w)* are then measured. Repeat the process for different window lengths,. Let *R/S(w)* be the average ratio *R(W)/S(w)*. en plot the logarithms of *R/S(w)* versus the logarithms of windows size *w*.

$$R/S(w) =R(W)/S(w)=w^{H} \qquad (4)$$

For a self-affine process, this plot follows a straight line whose slope equals the Hurst exponent *H*. The fractal dimension of *R/S* analysis D_{RS} is then calculated through the relationship, from (3).

$$D_{RS}=2-H \qquad (5)$$

Fractal Dimension estimation through Power Spectrum Analysis

Power spectrum method has been used extensively in literatures for pattern detection in different types of signals in physics and mathematics since 17th century [19], [20].

Power spectrum of a time series is expressed as the square of the amplitude of its Fourier transform and practically is a way to express the variance of the time series at different temporal scales. The power spectral density *E(f)* of a time series has a power-law relationship with time (frequency) and is given by:

$$E(f)\ \alpha\ f^{-B} \quad (6)$$

Where *f* is frequency and *f=1/t* (where *t* is time). The exponent is the slope of regression line and it is related to Hurst exponent, $H_{PS:fBm}$, and fractal dimension, $D_{PS:fBm}$, through:

$$H_{PS:fGn}=(B-1)/2 \quad \& \quad D_{PS:fBn}=(5-B)/2 \quad (7), (8)$$

For a *fGn* processes,

$$H_{PS:fGn}=(B+1)/2 \quad \& \quad D_{PS:fBn}=(3-B)/2 \quad (9), (10)$$

Where $H_{PS:fGn}$ and $D_{PS:fGn}$ are Hurst exponent and fractal dimension, respectively. For a stationary fractional Gaussian noise *-1< <1*, where as for a non-stationary fractional Brownian motion time series, *1< <3* . Therefore, a fractional Brownian motion and a fractional Gaussian noise characterized by the same Hurst exponent have different spectral exponents:

$$_{fBm}= \ _{fGn}-2 \quad (11)$$

There is a subtle critical fact that put power spectrum analysis ahead of rescaled range analysis in analyzing an unknown self-affine process. exponent covers a wider range of processes than *H* exponent. Therefore, finding the exponent of time series before estimating the *H* exponent and fractal dimension would avoid estimation errors.

Many literatures have understated this serious issue on analyzing self-affine processes. A logical structured approach for fractal analyzing of a self-affine time series is:

1. Verifying the power-law behavior of process and estimate its exponent by power spectrum analysis;
2. *1< <3* shows a non-stationary *fBm* process. Transform this process to a stationary *fGn* before further analysis.
3. In a self-affine time series *fBm* and *fGn* share the same *H* exponent with different exponent.
4. For a self-affine process with *-1< <1*, the *H* exponent and fractal dimension are calculated through *R/S* method.
5. Final analysis reveals the persistency or anti-persistency characteristic of the financial time series understudy, according to estimated fractal dimension.

The above structural method is currently one of the most logical approaches for fractal analysis of self-affine processes and avoids common ambiguities in many literatures.
This study applies the structured methodology on fractal analysis of Malaysian stocks market. The accuracy of persistence characteristics of individual stocks is verified by comparing the results with Zip's law observational method. This comparison expresses the quality of the applied methodology on fractal analysis of Malaysian stocks.

Zipf's law as an observational tool on determining persistent and anti-persistent processes

Zipf's law is another instance of power-law based on what Zipf [21] is termed as "principal of less effort". It states that the observation of the frequency of occurrences of any events has a power-law relationship with the rank of event; in which $r=1$ and $r=n$ denote the ranks for the least and the most frequent events respectively. More specifically, it states that the frequency of any occurrence is inversely proportional to its rank r as:

$$f_r=f_1/r \quad (12)$$

where f_r and f_1 are the frequency of *rth* largest occurrences and the most frequent event respectively. Zipf's law can be represented by regressing the log of frequency of occurrence as a function of rank, which leads to a power-law with a slope close to unity. Zipf's law can be generalized as:

$$f_r = f_1 / r^{\alpha} \quad (13)$$

where the log-log regression plot can be linear with any slope. More generally it can be written as:

$$X_r \, \alpha \, r^{-\alpha} \quad (14)$$

where X_r is value of any random variable with rank r and $\alpha=1$ for Zipf's law, or $\alpha \neq 1$ generalized Zipf's law. Equation (14) maybe applied on any continuous process. For a random process (that is, Brownian motion), Zipf's law appears linear in linear plot and in log-log plot it does not show any power-law behavior as expected from (14). For a Brownian motion on log-log plot, it produces a continuous roll-off from horizontal line (that is, $\alpha=0$) to a vertical line (that is, $\alpha=\infty$), that indicates no value is more probable than any other value in a random process.These observations are different in the case of fBm process. Persistent and anti-persistent behaviors in fBm produce a certain range of values before moving off gradually to another range of values. In an anti-persistent fBm the quantity of values between transitions is more gradual and contains more points than in a persistent fBm. In practice, use both the linear and logarithmic plots because the specific structural features of the Zipf's plot may partially hide compared to noise by the scale expansion related to log-log plots. Any step in a Zipf's plot is indicative of structural discontinuities within data.

Data

We consider the daily closing prices reported by Kuala Lumpur Stock Exchange (KLSE), extracted through Yahoo Finance website. The selected stocks are 12 stocks from most well performed Malaysian firms in different sectors ranked and reported by Bloomberg in 2011.The sample period is 2003:M1 to 2010:M12, yielding 2050 observations on each stock. The number of observations for each variable is equal and long enough to produce reliable and comparable results. To avoid any non-stationarity due to changes in mean of data over time, this data is adjusted for stock splits. Failure in adjustment would have showed discontinuities which are unrelated to any fundamentals and results in biased findings in fractal analysis [22], [23].

Results

Following the suggested structured methodology of fractal analysis, β of each stock is first calculated through a power spectrum analysis as listed in Table 1.

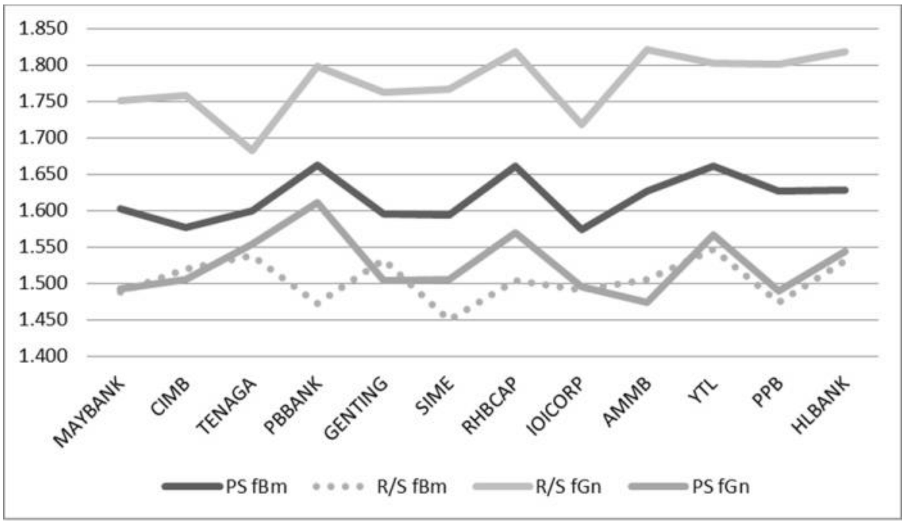
Table 1: β of stocks with power spectrum analysis

	May Bank	CIMB	TEN AG A	PB BAN K	GEN TIN G	SIME	RHBCA P	IOICOR P	AMMB	YTL	PPB	HL BANK
Ps fBm B	1.79	1.84	1.79	1.67	1.80	1.81	1.67	1.85	1.74	1.67	1.74	1.74
Ps fGm B	0.01	-0.01	-0.10	-0.22	-0.00	-0.01	-0.14	0.01	0.05	-0.13	0.02	-0.08
Fractal Dimensions												
Ps fBm	1.60	1.57	1.60	1.62	1.55	1.59	1.66	1.57	1.62	1.66	1.62	1.62
Ps fGm	1.49	1.50	1.55	1.61	1.50	1.50	1.57	1.49	1.47	1.56	1.48	1.54
R/S fBm B	1.48	1.52	1.53	1.47	1.53	1.44	1.50	1.49	1.50	1.54	1.47	1.52
R/S fGm B	1.75	1.75	1.68	1.79	1.76	1.76	1.81	1.71	1.82	1.80	1.80	1.81

The result shows *1<   <3* for all the variables, which categorize all the time-series considered as fractional Brownian motion (*fBm*) processes. This finding suggests a transformation of data from fBm to fGn in the second step prior to applying any other fractal analysis method. However, to have a brighter observation and to examine the accuracy of the suggested methodology, we apply *R/S* analysis on both *fBm* and *fGn* (that is, first difference transform of *fBm*). Fractal dimensions obtained from *R/S* analysis on *fBm* and *fGn*, together with fractal dimensions estimated by power spectrum analysis on *fBm* and *fGn*, are also listed in Table 1.

The estimated fractal dimensions for PS on fBm and *R/S* on *fGn* methods on all considered stocks are $1.5<D<2$, which exhibits anti-persistent behavior for all the stocks but with a different level of roughness. However, application of *PS* on *fGn* and direct application of *R/S* method on these stocks (that is, *R/S* on *fBm*) produce $1.4<D<1.6$, which exhibit slight persistency to anti-persistent behaviors. These estimated fractal dimensions via different methods are displayed in Fig. 1.

Fig 1: Estimated fractal dimensions via different methods

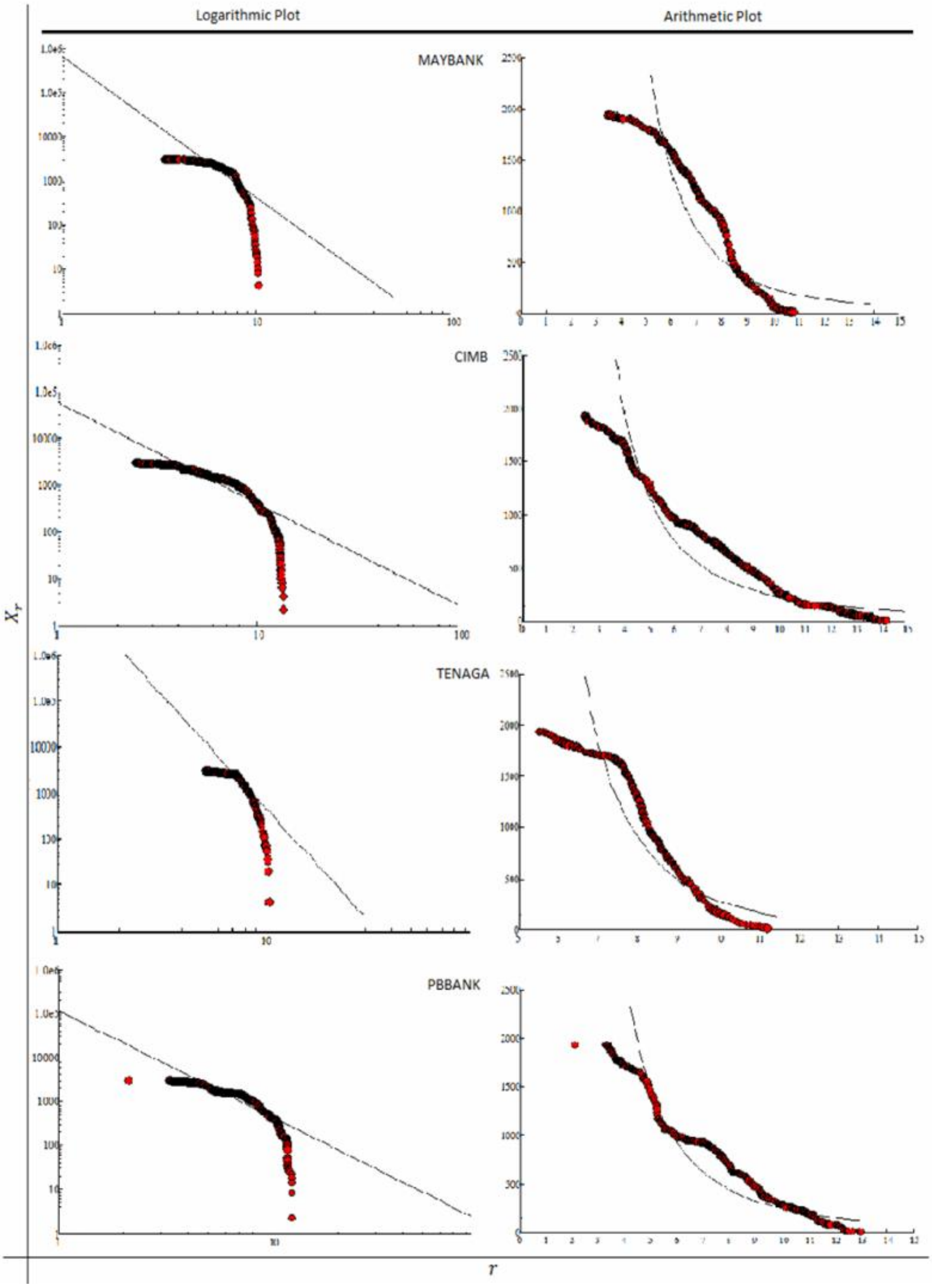


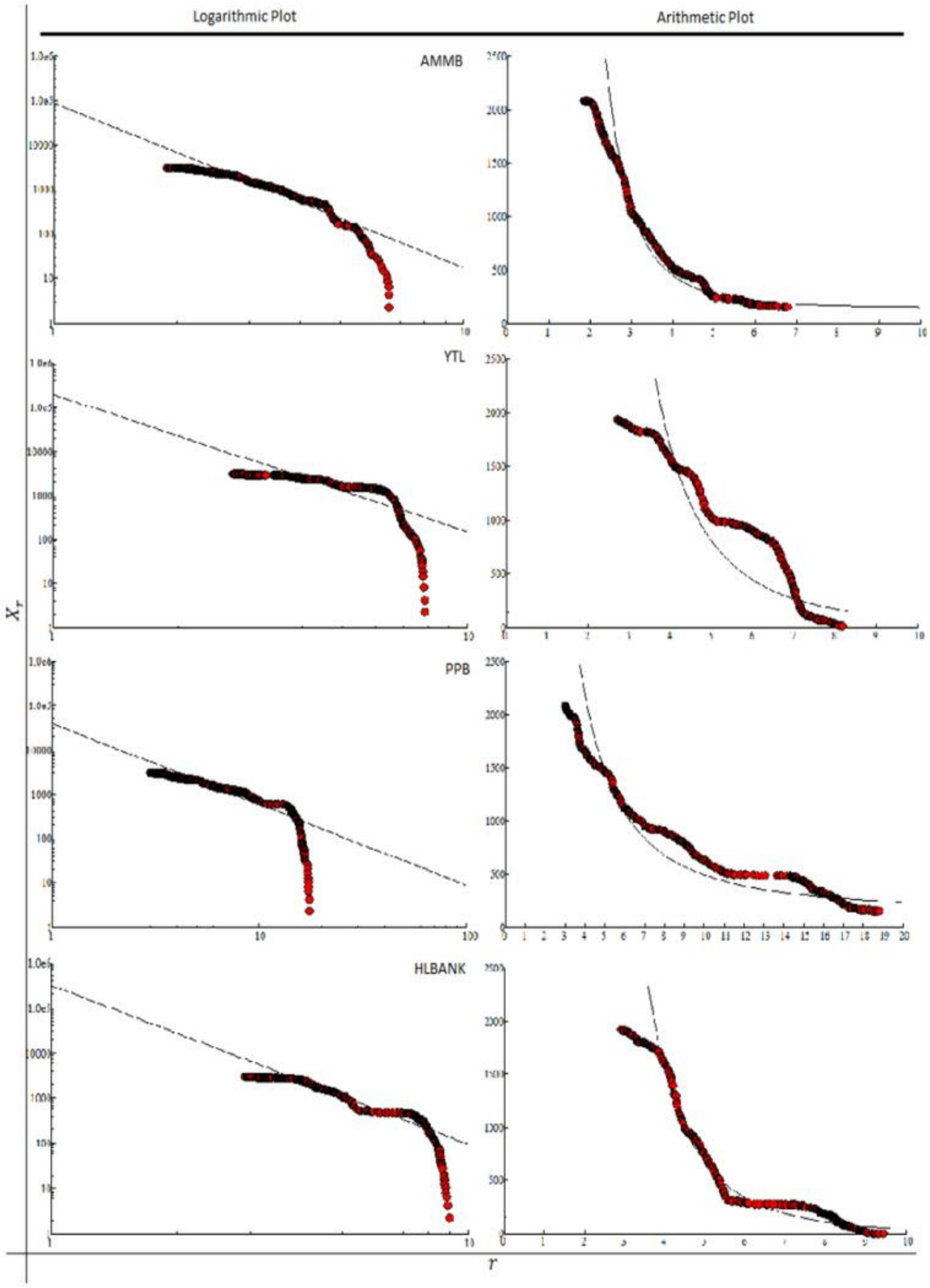
Notice that the fractal dimensions estimated through methods of *R/S* on *fGn*, and power spectrum on both *fBm* and *fGN* display similar trend across the stocks (that is, similar ups and downs for different stocks but with a shift in fractal dimension amount). This similarity could be evidence on the methods' accuracy.

Nevertheless, fractal dimensions estimated by these three methods (that is, power spectrum on *fBm* and *fGn*, and *R/S* on (*fGn*) belong to different ranges of deviation which characterizes the related stocks (that is, classified persistency of related stocks). The *R/S* on *fGn* and *PS* on *fGn* produce fractal dimensions in the range of $1.6<D<1.9$ and $1.5<D<1.7$ respectively, representing anti-persistent *fBm* of $1.4<D<1.7$, representing both persistent and anti-persistent behaviors for the different stocks. Applying any of these three methods fairly demonstrates a common relative change in roughness of the stocks, yet it creates vagueness in categorizing the stocks' persistency. Further analysis of designated stocks, through evaluation of the persistent behavior through plots of Zipf's law distribution of data, could overcome this ambiguity toward the selection of the most appropriate method among these three methods.

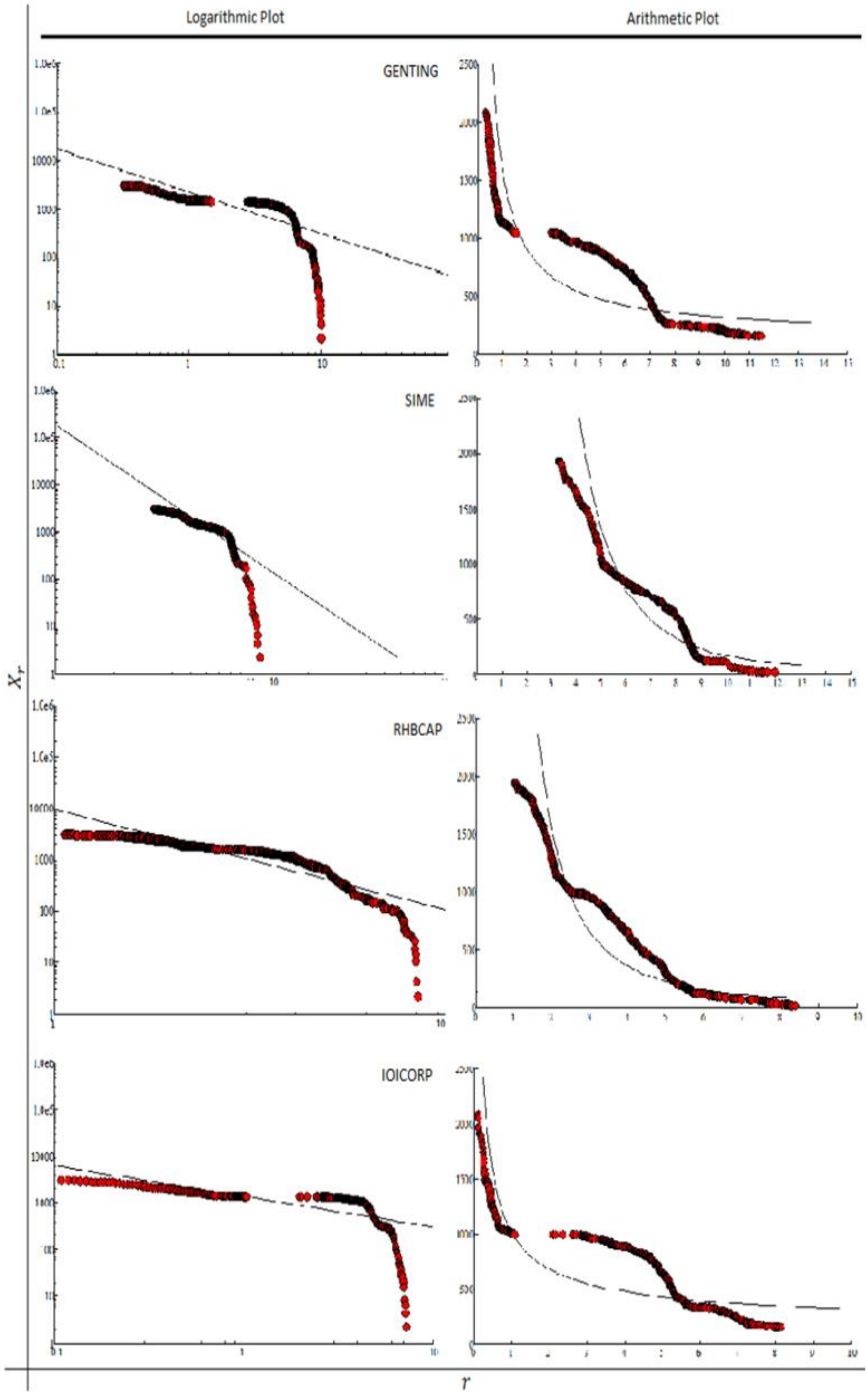
Fig. 2 exhibit linear and logarithmic plots of Zipf's law, for all the 12 stocks.

Figure 2: Linear and logarithmic Zipf's plots for all the 12 stocks





Logarithmic Plot
Arithmetic Plot
AMMB
YTL
PPB
HLBANK
X_r
r


Logarithmic Plot
Arithmetic Plot
GENTING
SIME
RHBCAP
IOICORP
X_r
r

Zipf's plots represent persistent behavior for MAYBANK, CIMB and TENAGA (Fig.2). The sharp transition from one range of values to another range is a particular feature of persistent behavior which is obvious in Zipf's plots of these first three stocks. For PBBANK, we observe more gradual transition between high and low values with more concentration of points at each transition curve, which is a particular feature of anti-persistent behavior. GENTING and IOICORP (Fig.3) produce similar plots including a gap in the plots due to discontinuities of data sets. The sharp fall in market return for both of these stocks has affected their structural behavior severely. Thus, neither fractal dimension nor Zipf's plots are helpful on stating the persistent behavior for these two stocks. Zipf's plot of SIME (Fig.3) illustrates sharp upward and downward roll-offs to exemplify a persistent characteristic; yet abundance of points in a few transition curves, which add to a little anti-persistent behavior of the stock, reduces its persistent characteristic toward randomness. For RHBCAP (Fig. 3), although the arithmetic plot may look like the arithmetic plot of SIME (Fig. 3), richness of points in turning curves of RHBCAP (Fig. 3), which is clearly observable through both the linear and log-log plots, indicates strong anti-persistent behavior of this stock. Following the above procedure on the analysis of the Zipf's plot of stocks as illustrated in Fig.4, more anti-persistent than persistent characteristic is observable for AMMB, YTL, PPB and HLBANK (Fig. 4).

Conclusion

The findings from this study emphasize the subtleness of applying fractal analysis methods on financial data. Sensitivity is particularly higher in order to extract persistent or anti-persistent characteristic of series. Although all methods, which categorized the stocks as fractional Brownian processes, separates them from randomness (that is, Brownian motion) accurately, applying the same methods for further analysis to the data in classifying them as persistent or antipersistent, should be done with more thoughtfulness. This study has revealed the discrepancy in results on detecting the persistent and anti-persistent fractal behavior by applying different analysis methods. It has been identified that even proposed structured fractal analysis method could not produce consistent results. The application of less used tools such as Zipf's plots on fractal analyzing of financial data along with common fractal analysis tools produce a better judgment on characterizing market behavior. Further similar studies on different financial data sets by applying more diverse fractal dimension estimation methods may end in choosing the best fractal analyzing procedure, reducing the ambiguity of results on this domain.

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